Off-Site Instruction Packet Day 5

6-3 Day 2 Angles and Radian Measure

Arc Length and Angular Speed

The formula for arc length can also be written in terms of radians.

Arc Length

An arc with central angle measure θ radians has length

$$l = r\theta$$

In other words, the arc length is the radius times the radian measure of the central angle of the arc.

Example 5: **Arc Length**

The second hand on a clock is 6 inches long. How far does the tip of the hand move in 15 seconds? *15 seconds* $= \frac{1}{4}$ *of a revolution*

$$\frac{1}{4} \cdot 2\pi = \frac{\pi}{2} \qquad l = 6\left(\frac{\pi}{2}\right) = 3\pi = 9.4\ inches$$

Example 6: **Central Angle Measure**

Find the central angle measure (in radians) of an arc of length 5 cm on a circle with a radius of 3 cm.

$$5 = 3\theta \qquad \theta = \frac{5}{3}$$

Linear and Angular Speed

Suppose that a wheel is rotating at a constant rate around its center, O, and P is a point on the outer edge of the wheel. There are two ways to measure the speed of point P, in terms of the distance traveled in terms of the angle of rotation. The two measures of speed are called *linear speed* and *angular speed*. *(d=rt)*

Recall that speed of a moving object is ____$\frac{distance}{time}$_____. If the object is traveling in a circular path with radius r, the **linear speed** is given by

$$linear\ speed = \frac{arc\ length}{time} = \frac{r\theta}{t}$$

and the **angular speed** is given by

$$angular\ speed = \frac{angle}{time} = \frac{\theta}{t}$$

where θ is the radian measure of the angle through which the object traveled in time t. Notice the relationship between linear speed and angular speed:

$$linear\ speed = r \cdot angular\ speed$$

Example 7: **Linear and Angular Speed**

A merry-go-round makes 8 revolutions per minute.

a) What is the angular speed of the merry-go-round in radians per minute?

$$8 \cdot 2\pi = 16\pi$$

$$\frac{\theta}{t} = \frac{16\pi}{1} \qquad 16\pi\ radians\ per\ minute$$

b) How fast is a horse 12 feet from the center traveling?

$$linear\ speed = r \cdot angular\ speed$$

$$= 12 \cdot 16\pi = 192\pi\ ft/min \approx 6.9 mph$$

c) How fast is a horse 4 feet from the center traveling?

$$4 \cdot 16\pi = 64\pi \frac{ft}{min} \approx 2.3\ mph$$

Assessment:

Pg 441 #49-55 (odd)