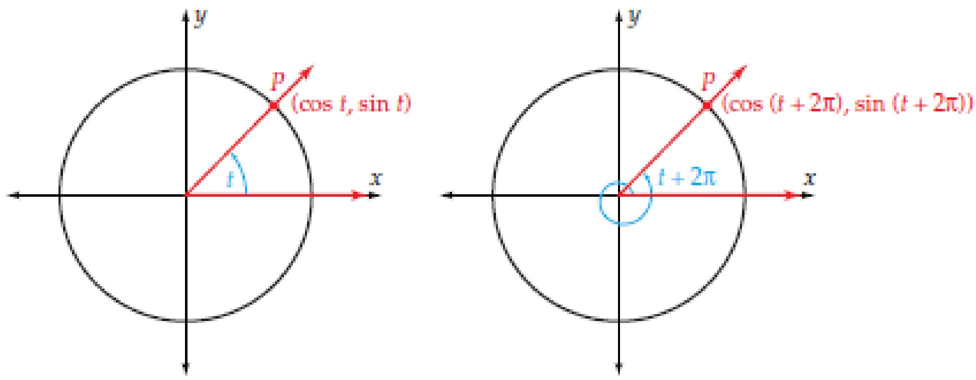
Off-Site Learning Packet Day 9

6-5 Day 2 **Basic Trigonometric Identities**

Periodicity Identities

Let t be any real number. Construct two angles in standard position of measure t and $t + 2\pi$ radians



In both cases, the sine is the y-coordinate of P, so

$$\sin t = \sin(t + 2\pi)$$

In addition, the terminal side of the angle is the same for measures of t, $t \pm 2\pi$, $t \pm 4\pi$, $t \pm 6\pi$, and so on. Thus,

$$\sin t = \sin(t \pm 2\pi) = \sin(t \pm 4\pi) = \sin(t \pm 6\pi) = \cdots$$

Similarly in both cases, the cosine is the x-coordinate of P, so

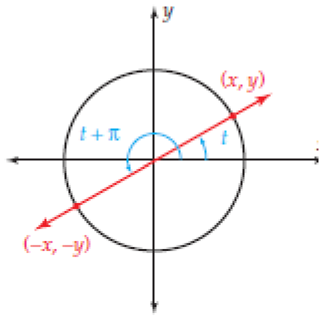
$$\cos t = \cos(t \pm 2\pi) = \cos(t \pm 4\pi) = \cos(t \pm 6\pi) = \cdots$$

Periodic Function – *a nonconstant function that repeats its values at regular intervals; a function f for which there exists some constant k such that $f(t) = f(t + k)$ for every number t of the domain f*

Period (of a function) – *the smallest value of k in a function f for which there exists some constant k such that $f(t) = f(t + k)$ for every number t of the domain f*

Since the tangent function is the quotient of the sine and cosine functions, it must also be true that ________$\tan t = \tan(t + 2\pi)$_________________.

However, there is a number smaller than 2π that has this property. The figure below shows the angles t and $t + \pi$.



A rotation of π radians is the same as a rotation of $180°$, so the image of point (x, y) is _______$(-x, -y)$____________. Thus,

$$\mathbf{\tan(t + \pi) = \frac{-y}{-x} = \frac{y}{x} = \tan t}$$

Periodicity Identities

The sine and cosine functions are periodic with period 2π. For every real number t,

$$\mathbf{\sin(t \pm 2\pi) = \sin t} \quad \textbf{and} \quad \mathbf{\cos(t \pm 2\pi) = \cos t}$$

The tangent function is periodic with period π. For every number t is the domain of the tangent function,

$$\mathbf{\tan(t \pm \pi) = \tan t}$$

Example 4: **Periodicity Identities**

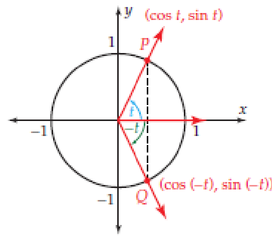
Find the exact value of $\sin \frac{13\pi}{6}$

Method 1: $\sin \frac{13\pi}{6} = \sin\left(\frac{\pi}{6} + \frac{12\pi}{6}\right) = \sin\left(\frac{\pi}{6} + 2\pi\right) = \sin \frac{\pi}{6} = \frac{1}{2}$

Method 2: Graph $\frac{13\pi}{6}$ and make a reference triangle. Find sine of reference angle

Negative Angle Identities

Let t be any real number and construct two angles in standard position of measure t and $-t$ radians, as shown in the figure.



Since the point Q is the reflection of the point P across the x-axis, the x-coordinates of P and Q are the same, and the y-coordiantes are opposites of each other. Thus,

$$\mathbf{\cos t = \cos(-t)} \qquad \textbf{and} \qquad \mathbf{\sin t = -\sin(-t)}$$

Also,

$$\mathbf{\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin t}{\cos t} = -\frac{\sin t}{\cos t} = -\tan t}$$

Negative Angle Identities

$$\sin(-t) = -\sin t$$

$$\cos(-t) = \cos t$$

$$\tan(-t) = -\tan t$$

Example 5: **Negative Angle Identities**

Find the exact value of $\sin -\frac{\pi}{6}$ and of $\cos\left(-\frac{\pi}{6}\right)$

Method 1: $\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$ $\qquad$ $\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Method 2: Graph $-\frac{\pi}{6}$ and make a reference triangle. Find sine and cosine of reference angle

Assessment:

Pg 461 #27-35 (odds)