Off-Site Learning Packet Day 3

Fractional Expressions

A fraction is in ____***lowest terms***_____ if its ___***denominator***_____ and ____***numerator***_____ have no common factors except for ± 1. To express a fraction in lowest terms, factor numerator and denominator and cancel common factors.

Example 1:

$$\frac{x^2+x-6}{x^2-3x+2}$$

$$\frac{(x+3)(x-2)}{(x-2)(x-1)} = \frac{x+3}{x-1}$$

To add fractions with the same denominator, simply add the numerators as in ordinary arithmetic: $\frac{a}{b} + \frac{c}{b} = \quad \boldsymbol{\frac{a+c}{b}}$
Subtraction is done similarly.

Example 2:

$$\frac{7x^2+2}{x^2+3} - \frac{4x^2+2x-5}{x^2+3}$$

$$\frac{7x^2+2-4x^2-2x+5}{x^2+3} = \frac{3x^2-2x+7}{x^2+3}$$

To add or subtract fractions with different denominators, you must first find a common denominator.

Example 3:

$$\frac{2x+1}{3x} - \frac{x^2-2}{x-1} \qquad \text{LCD: } 3x(x-1)$$

$$\frac{(x-1)(2x+1)}{3x(x-1)} - \frac{3x(x^2-2)}{3x(x-1)} = \frac{2x^2+x-2x-1-3x^3+6x}{3x(x-1)} = \frac{-3x^3+2x^2+5x-1}{3x(x-1)}$$

Although the product of the denominators can always be used as a common denominator, it's often more efficient to use the *least common denominator*. The least common denominator can be found by factoring each denominator completely and then taking the product of the highest power of each of the distinct factors.

Example 4:

$\frac{1}{100}+\frac{1}{120}$ LCD: 600 *Use calculator: MATH > 8:lcm*

$\frac{6}{600}+\frac{5}{600}=\frac{11}{600}$

Example 5:

Find the least common denominator

$$\frac{1}{x^2+2x+1},\ \frac{5x}{x^2-x},\ \frac{3x-7}{x^4+x^3}$$

Factor all denominators: $(x+1)^2, x(x-1);\ x^3(x+1)$

LCD: $x^3(x-1)(x+1)^2$

To express one of several fractions in terms of the least common denominator, multiply its numerator and denominator by those factors in the common denominator that *don't* appear in the denominator of the fraction.

Example 6:

Find the LCD and express each with the LCD

$$\frac{1}{(x+1)^2},\quad \frac{5x}{x(x+1)},\quad \frac{3x-7}{x^3(x+1)}$$

LCD: $x^3(x+1)^2$

$$\frac{x^3}{x^3(x+1)^2},\quad \frac{5x^3}{x^3(x+1)^2},\quad \frac{(3x-7)(x+1)}{x^3(x+1)^2}$$

Example 7: **Simplify**

$\frac{1}{z}+\frac{3z}{z+1}-\frac{z^2}{(z+1)^2}$ LCD: $z(z+1)^2$

$\frac{(z+1)^2}{z(z+1)^2}+\frac{3z(z)(z+1)}{z(z+1)^2}-\frac{z^3}{z(z+1)^2}$

$\frac{z^2+2z+1}{z(z+1)^2}+\frac{3z^3+3z^2}{z(z+1)^2}-\frac{z^3}{z(z+1)^2}$

$$\frac{2z^3+4z^2+2z+1}{z(z+1)^2}$$

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Simplify each expression.

1. $\frac{x-5}{x^2-7x+10}$

2. $\frac{r-2}{5r-10}$

3. $\frac{r^2+3r-70}{7r^3-49r^2}$

4. $\frac{56n-8}{16n-64}$

5. $\frac{7a^2-75a+50}{9a^2-72a-180}$

6. $\frac{9p^2-99p+162}{10p^2+23p+6}$

7. $\frac{7}{5} \cdot \frac{16n^2}{14}$

8. $\frac{11x}{16} \cdot \frac{12}{11x}$

9. $\frac{x+1}{x^2+10x+24} \div \frac{1}{x+4}$

10. $\frac{n+3}{25n-5n^2} \div \frac{3}{5n^2-25n}$

11. $\frac{7x-4}{49x-70} \cdot \frac{21x-30}{63x-36}$

12. $\frac{7n^2-60n+32}{40n+40} \div \frac{7n-4}{40n+40}$

13. $\frac{u-5v}{8u^2} - \frac{u+2v}{8u^2}$

14. $\frac{a+b}{24a} + \frac{a-3b}{24a}$

15. $\frac{7}{4} + \frac{5x}{3x-24}$

16. $\frac{5}{3} + \frac{7n+7}{n+3}$

17. $\frac{5}{3} + \frac{a-4}{8a^2-28a+20}$