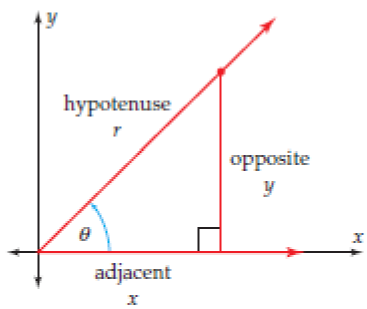
Off-Site Learning Packet Day 6

6-4 Day 1 **Trigonometric Functions**

Trigonometric Ratios were defined for acute angles in Section 6.1. The next step is to develop a definition of these ratios that applies to angles of any measure.

To do this, first consider an acute angle θ in standard position. Choose a point P, with coordinates (x, y), on the terminal side, and draw a right triangle.



$$x^2 + y^2 = r^2$$

Trigonometric Ratios in the Coordinate Plane

Let θ be an angle in standard position and let $P(x, y)$ be any point on the terminal side of θ. Let r be the distance from (x, y) to the origin:

$$r = \sqrt{x^2 + y^2}$$

Then the trigonometric ratios are defined as follows:

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

Example 1: **Trigonometric Ratios in the Coordinate Plane**

Find the sine, cosine, and tangent of the angle θ, whose terminal side passes through the point $(-3, -2)$.

$$x = -3 \qquad y = -2 \qquad r = \sqrt{(-3)^2 + (-2)^2} = \sqrt{13}$$

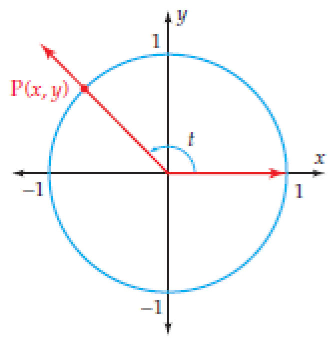
$$\sin\theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$$

$$\cos\theta = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$$

$$\tan\theta = \frac{-2}{-3} = \frac{2}{3}$$

<u>Trigonometric Functions and the Unit Circle</u>

<u>Unit Circle</u> – *the circle of radius 1 centered at the origin of the coordinate plane.*



The distance from P to the origin is 1 because the unit circle has radius 1. Using the point P, $r = 1$, and the definition of trigonometric functions of real numbers shows the following:

$$\sin t = \frac{y}{r} = \frac{y}{1} = y \qquad\qquad \cos t = \frac{x}{r} = \frac{x}{1} = x$$

<u>Unit Circle Description of Trigonometric Functions</u>

Let t be a real number and let P be the point where the terminal side of an angle of t radians in standard position meets the unit circle. Then

P has cooridnates $(\cos t, \sin t)$

and

$$\tan t = \frac{y}{x} = \frac{\sin t}{\cos t} \qquad\qquad \cot t = \frac{x}{y} = \frac{\cos t}{\sin t}$$

$$\sec t = \frac{1}{x} = \frac{1}{\cos t} \qquad\qquad \csc t = \frac{1}{y} = \frac{1}{\sin t}$$

Graphing Exploration

With your calculator in radian mode and parametric graphing mode, set the range (window) values as follows:

$0 \le t \le 2\pi$ $\quad\quad$ $-1.8 \le x \le 1.8$ $\quad\quad$ $-1.2 \le y \le 1.2$

Then, graph the curve given by these parametric equations:

$$x = \cos t \quad\quad\quad y = \sin t$$

The graph is the unit circle. Use the trace to move around the circle. At each point, the screen will display numbers: the values of t, x, and y. For each t, the cursor is on the point where the terminal side of an angle of t radians meets the unit circle. So the corresponding x is the number $\cos t$ and the corresponding y is the number $\sin t$.

Domain and Range

For any real number t, an appropriate angle of t radians and its intersection point with the unit circle are always defined, so

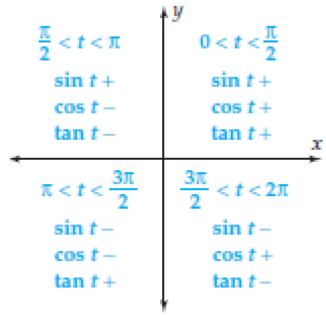
The domain of the sine function and of the cosine function is the set of real numbers.

The range of a function is the set of all possible outputs. Because $\sin t$ and $\cos t$ are the coordinates of a point on the unit circle, they take on all values between -1 and 1 and no other values. Thus,

The range of the sine function and of the cosine function is the set of real numbers between -1 *and* 1 $[-1, 1]$

Signs of the Trigonometric Functions

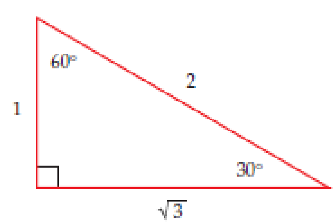
It is often important to know whether the value of the trigonometric function is positive or negative.

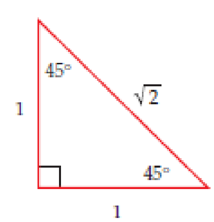


Exact Values of Trigonometric Functions

Although a calculator is used to evaluate trigonometric functions approximately, there are a few special numbers for which exact values can be found.

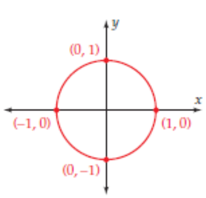
t	$\frac{\pi}{6}$ $(30°)$	$\frac{\pi}{4}$ $(45°)$	$\frac{\pi}{3}$ $(60°)$
$\sin t$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos t$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan t$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\frac{1}{1} = 1$	$\frac{\sqrt{3}}{1} = \sqrt{3}$
$\csc t$	$\frac{2}{1} = 2$	$\frac{\sqrt{2}}{1} = \sqrt{2}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
$\sec t$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\frac{\sqrt{2}}{1} = \sqrt{2}$	$\frac{2}{1} = 2$
$\cot t$	$\frac{\sqrt{3}}{1} = \sqrt{3}$	$\frac{1}{1} = 1$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$





Example 2: **Exact Values of Trigonometric Functions**

Find the exact value of the sine, cosine, and tangent functions when $t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2},$ and 2π.



t	$\sin t$	$\cos t$	$\tan t$
0	0	1	0
$\frac{\pi}{2}$	1	0	undefined
π	0	−1	0
$\frac{3\pi}{2}$	−1	0	undefined
2π	0	1	0

Assessment:

Pg 452 #1-13 (odd)