Questions

Question 1

The numeration system we use in our daily lives is called *base ten*, also called *decimal* or *denary*. What, exactly, does "base ten" mean?

Given the following base-ten number, identify which digits occupy the "one's place," "ten's place," "hundred's place," and "thousand's place," respectively:

$$5,183$$

file 01195

Question 2

Observe the following sequence of numbers:

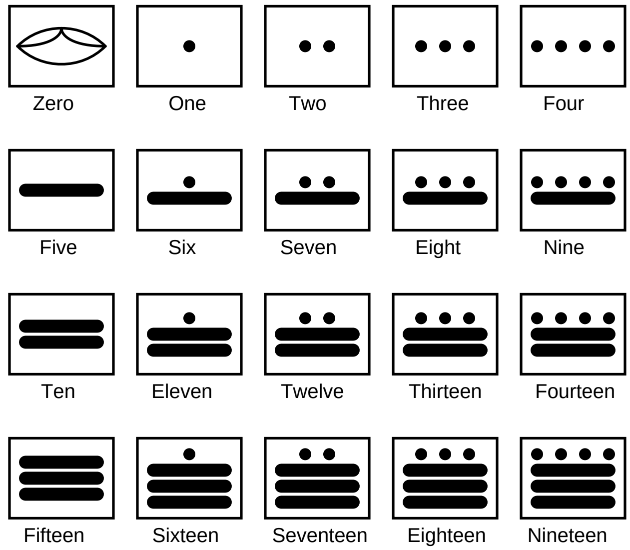
00
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
16
17
18
19
20
21
.
.
.

What pattern(s) do you notice in the digits, as we count upward from 0 to 21 (and beyond)? This may seem like a very simplistic question (and it is!), but it is important to recognize any inherent patterns so that you may understand counting sequences in numeration systems with bases other than ten.

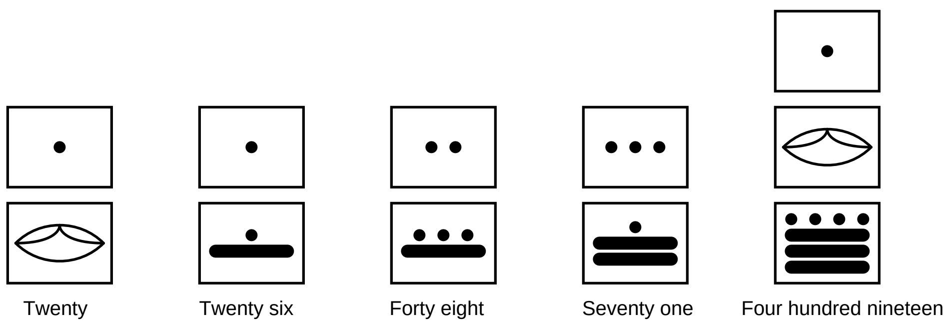
file 01196

Question 3

The ancient Mayans used a *vigesimal*, or base-twenty, numeration system in their mathematics. Each ”digit” was a actually a composite of dots and/or lines, as such:

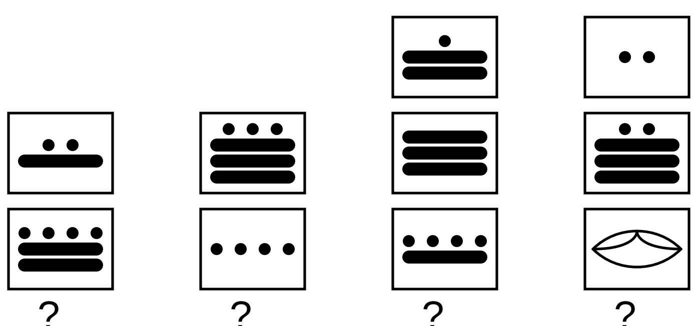


To represent numbers larger than twenty, the Mayans combined multiple ”digits” the same way we do to represent numbers larger than ten. For example:



Based on the examples shown above, determine the place-weighting of each ”digit” in the vigesimal numeration system. For example, in our denary, or base-ten, system, we have a one’s place, a ten’s place, a hundred’s place, and so on, each successive ”place” having ten times the ”weight” of the place before it. What are the values of the respective ”places” in the Mayan system?

Also, determine the values of these Mayan numbers:



file 01197

Question 4

Digital computers use a numeration system with a base of *two*, rather than a base of *ten* as we are accustomed to using. It is much easier to engineer circuitry that counts in "binary" than it is to design circuits that count in any other base system. Based on what you know of numeration systems, answer the following questions:

- How many different symbols (ciphers) are there in the binary numeration system?
- What are the different place-weight values in the binary system?
- How would you represent the number "seventeen" in binary?
- In our base-ten (denary) numeration system, each character is called a "digit." What is each character called in the binary numeration system?

file 01198

Question 5

If a digital meter display has four digits, it can represent any number from 0000 to 9999. This constitutes ten thousand unique numbers representable by the display. How many unique numbers could be represented by five digits? By six digits?

If an ancient Mayan ledger had spaces for writing numbers with three "digits" each, how many unique numbers could be represented in each space? What if the spaces were expanded to hold four "digits" each?

If a digital circuit has four *bits*, how many unique binary numbers can it represent? If we expanded its capabilities to eight bits, how many unique numbers could be represented by the circuit?

After answering these questions, do you see any mathematical pattern relating the number of "places" in a numeration system and the number of unique quantities that may be represented, given the "base" value ("radix") of the numeration system? Write a mathematical expression that solves for the number of unique quantities representable, given the "base" of the system and the number of "places".

file 01199

Question 6

What is the largest number that can be counted to, in a base-ten system with six digits? How about in a base-twenty (vigesimal) system with four places? How about in a base-two (binary) system with ten bits?

file 01200

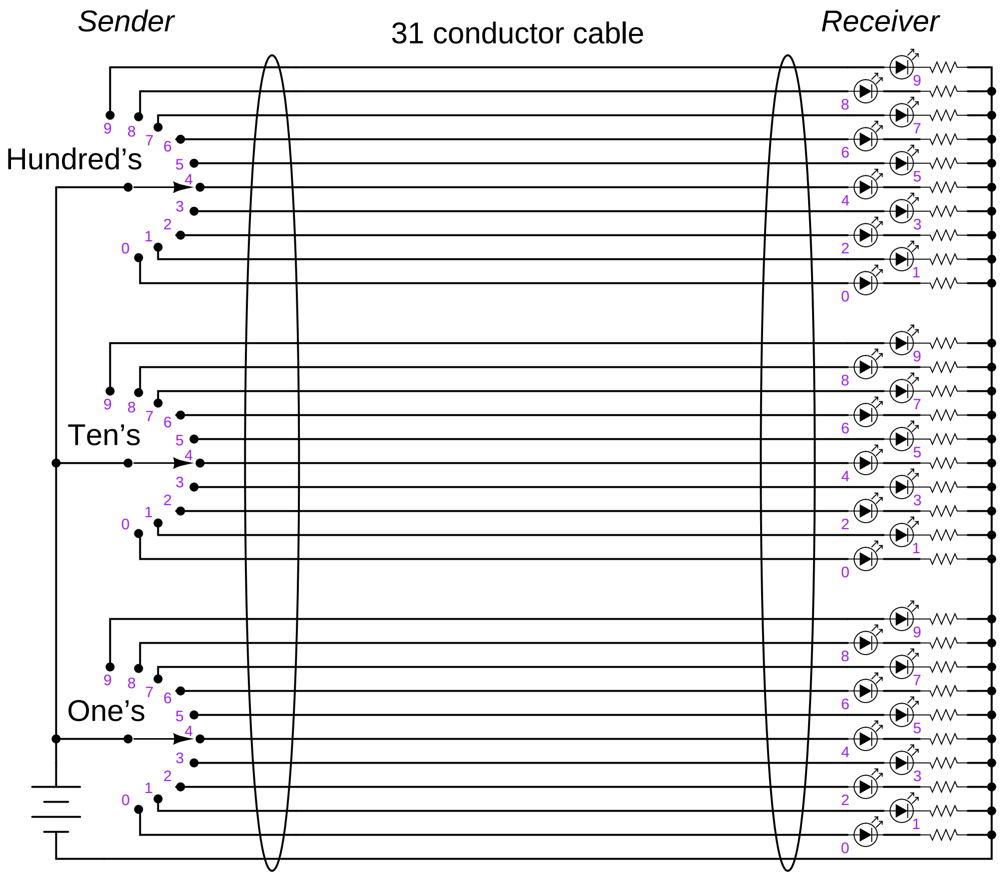
Question 7

How many binary bits are needed to count up to the number one million three hundred thousand seven hundred sixty two? Try to answer this question without converting this quantity into binary form, and then explain the mathematical procedure!

file 01228

Question 8

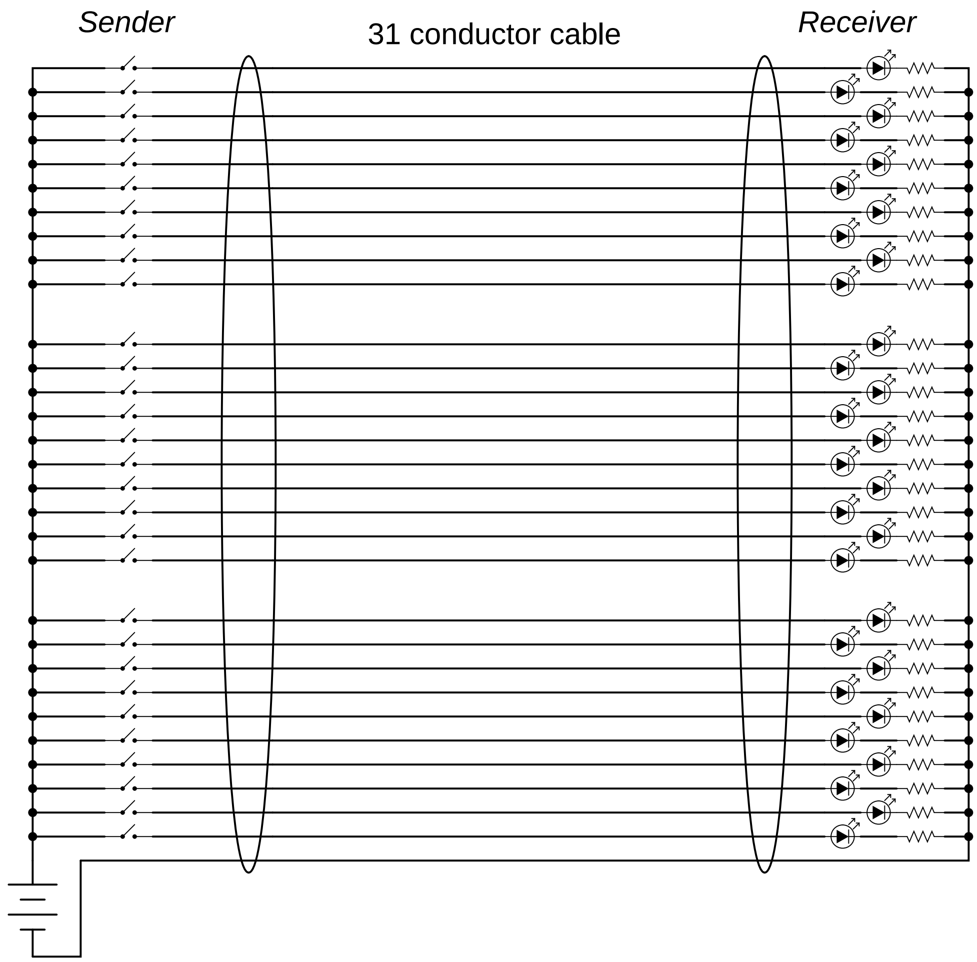
The circuit shown in this diagram is used to transmit a numerical value from one location to another, by means of switches and lights:



Given the switches and lights shown, any whole number between 0 and 999 may be transmitted from the switch location to the light location.

In fact, the arrangement shown here is not too different from an obsolete design of electronic base-ten indicators known as *Nixie tube* displays, where each digit was represented by a neon-filled glass tube in which one of ten distinct electrodes (each in the shape of a digit, 0-9) could be energized, providing glowing numerals for a person to view.

However, the system shown in the above diagram is somewhat wasteful of wiring. If we were to use the same thirty-one conductor cable, we could represent a much broader range of numbers if each conductor represented a distinct binary bit, and we used binary rather than base-ten for the numeration system:



How many unique numbers are representable in this simple communications system? Also, what is the greatest individual number which may be sent from the "Sender" location to the "Receiver" location?

file 01201

Question 9

Explain why binary is a natural numeration system for expressing numbers in electronic circuits. Why not decimal or some other base of numeration?

How do you suspect binary numbers may be *stored* in electronic systems, for future retrieval? What advantages are there to the use of binary numeration in storage systems?

file 01203